Valuation of a structured transaction

XXXIII Heidelberg Physics Graduate Days

Heidelberg, October 8th, 2014

Agenda

The transaction

A walk through

Background information

Some preliminary comments

- The case study is based on an actual project we have performed recently
- For confidentiality reasons, all numbers, names and details of the transaction and the assignment are pure fiction

- Purpose of the talk is to give some insight in
  - some real problems we face in our daily work
  - the methodologies applied
  - the required tools and skills

- If you have any questions during the talk, don’t hesitate to ask!

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Assignment

- Before the financial crisis, the client has participated in a set of similarly structured transactions
- Since then, the risk of (joint) default events and the importance of funding costs have significantly increased
- Therefore, the model in place is no longer seen to be adequate
- The task is to help the client implementing a new, improved model that
  - simulates all relevant risk factors
  - is able to model dependencies between different parties involved in the transaction
  - could be calibrated to liquid market quotes

The parties involved

Where the story begins

PTC

PTC: Public Transport Company

- Needs to raise money to build a new railway track
- Could be also any other utility company
- Typically owned by public community
- Provides basic infrastructure
- Making profits is not the major interest

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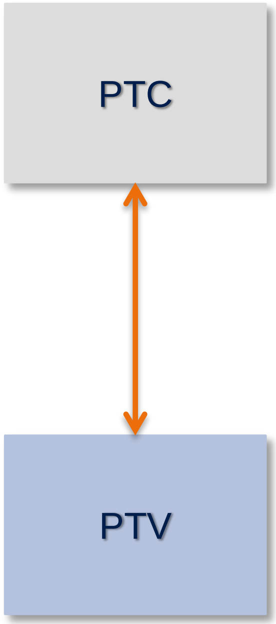
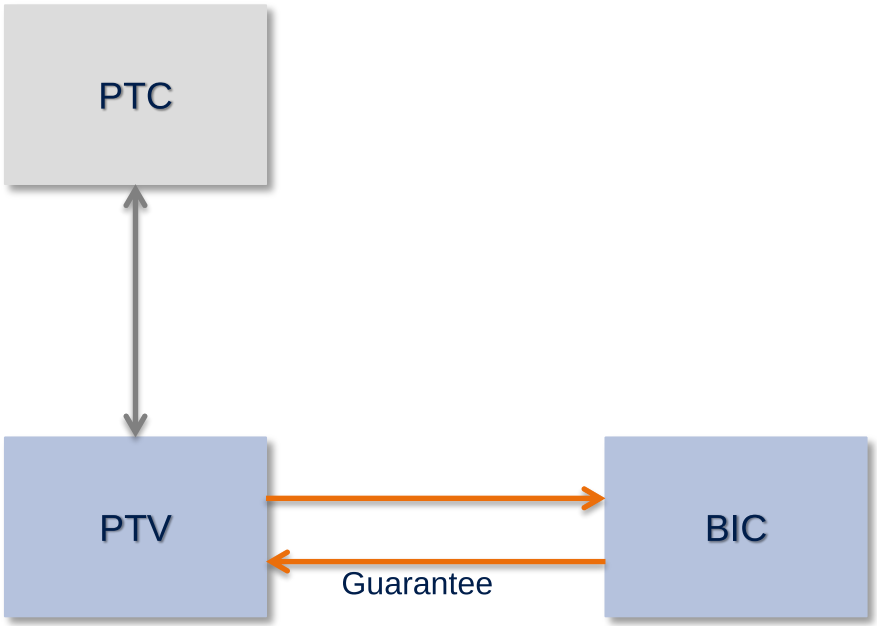
PTC is founding Special Purpose Vehicle (SPV) for funding purposes



PTV: Special purpose vehicle

- Fully owned by PTC
- Only purpose is to issue a structured bond for financing PTC‘s railway project
- Safeguard against trouble: if project fails, PTV may go bankrupt without affecting PTC (too much)

BIC helps to increase attractiveness of bond investment



BIC: Bond Insurance Company

- Guarantor of bond issued by PTV
- Replaces bond payments if PTV defaults
- Often monoline insurance company
- Monoliner reduces risk by diversifying over many clients
- Guarantee increases credit quality and reduces risk of bond holder

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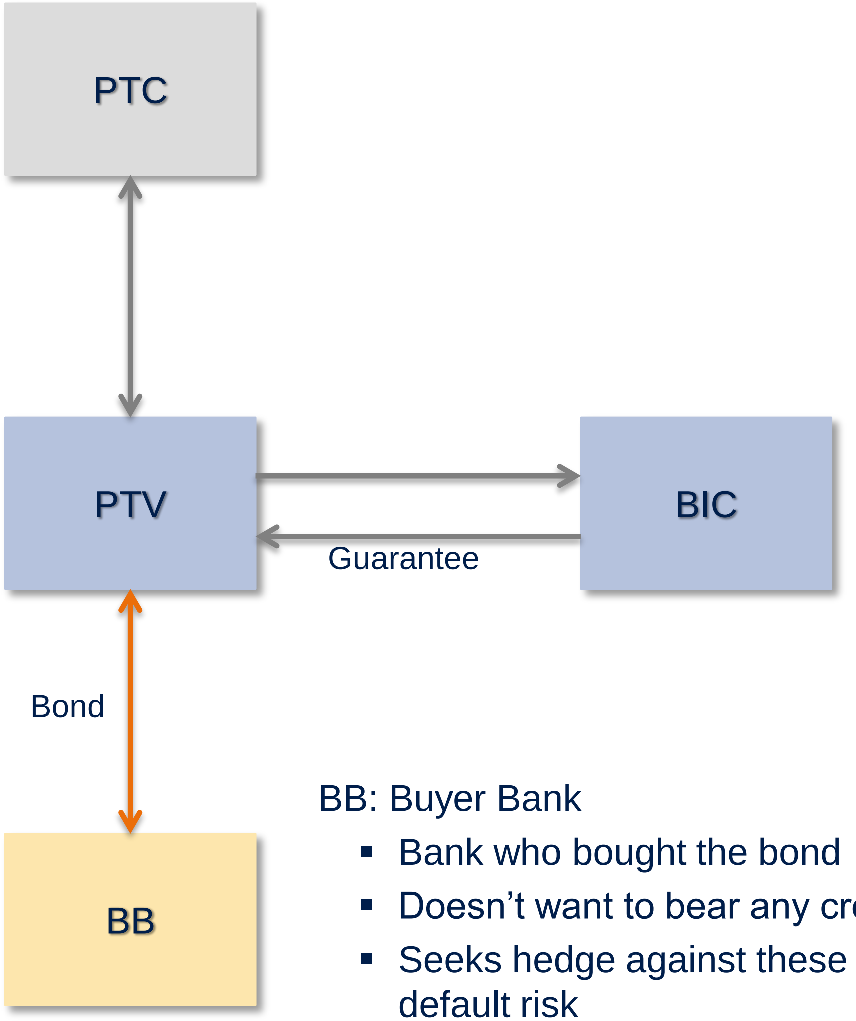
Someone has to buy the issued bonds



BB: Buyer Bank

- Bank who bought the bond
- Doesn’t want to bear any credit or market risk
- Seeks hedge against these risks to construct synthetic floater + spread without default risk

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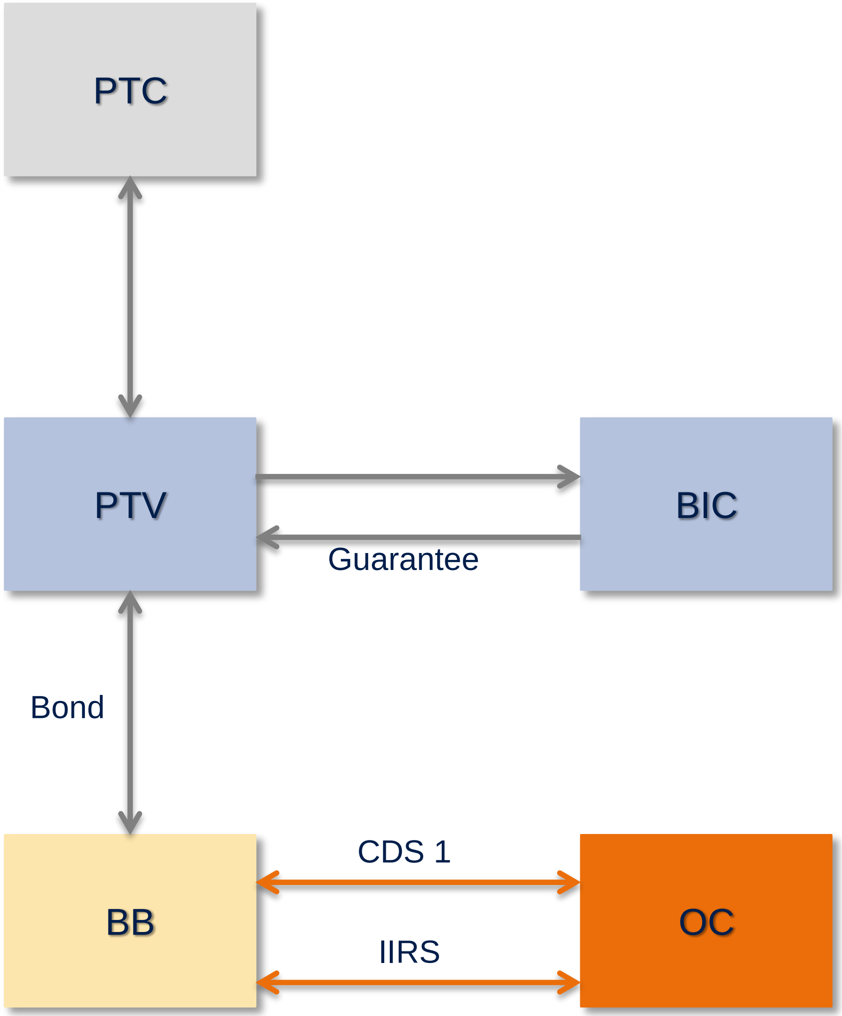
Our client enters the scene



OC: Hedging partner of BB

- Our client
- Sold protection against default of PTV/BIC and market risk to BB

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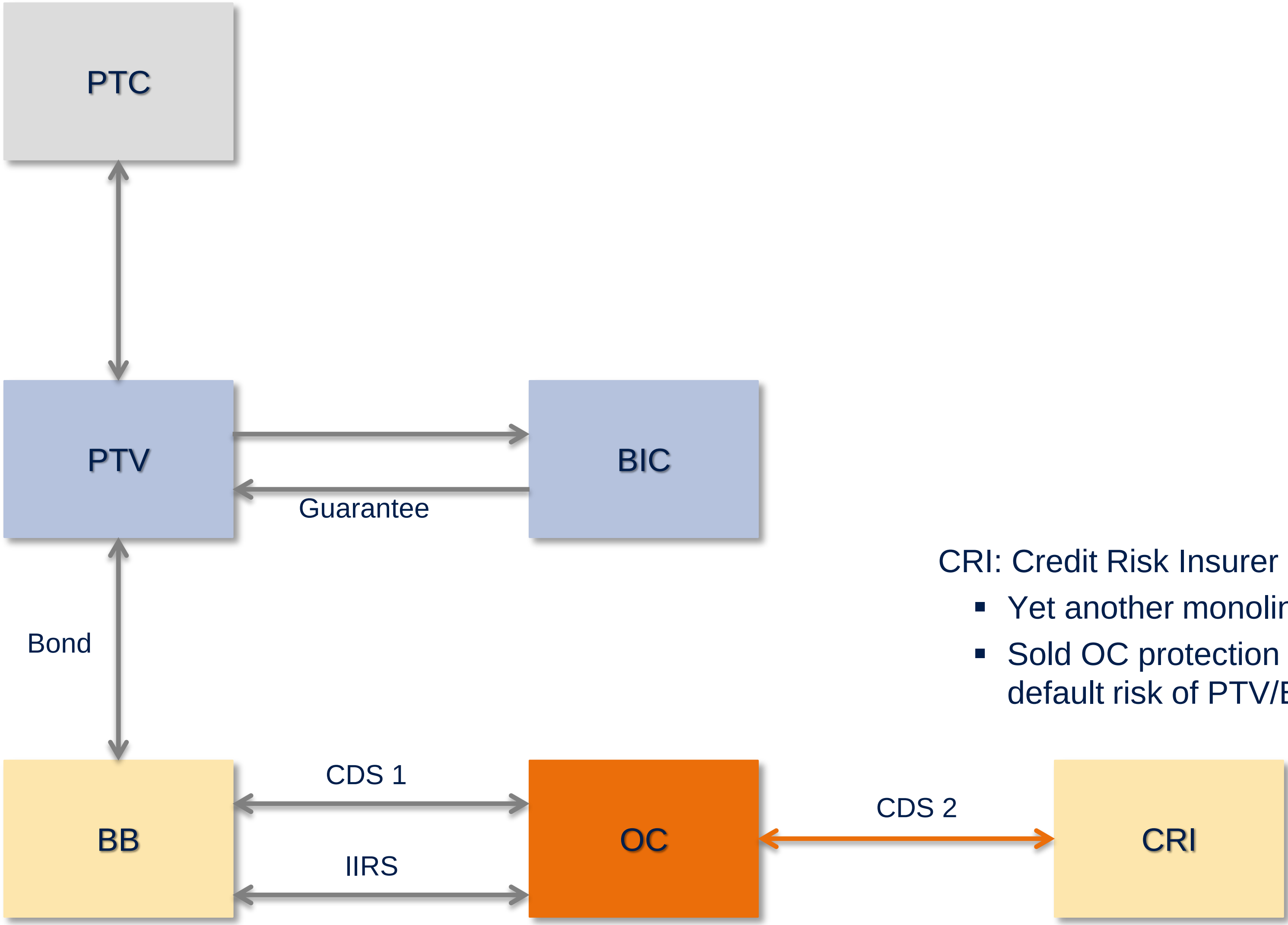
OC wants to be on the safe side



CRI: Credit Risk Insurer

- Yet another monoliner
- Sold OC protection against default risk of PTV/BIC

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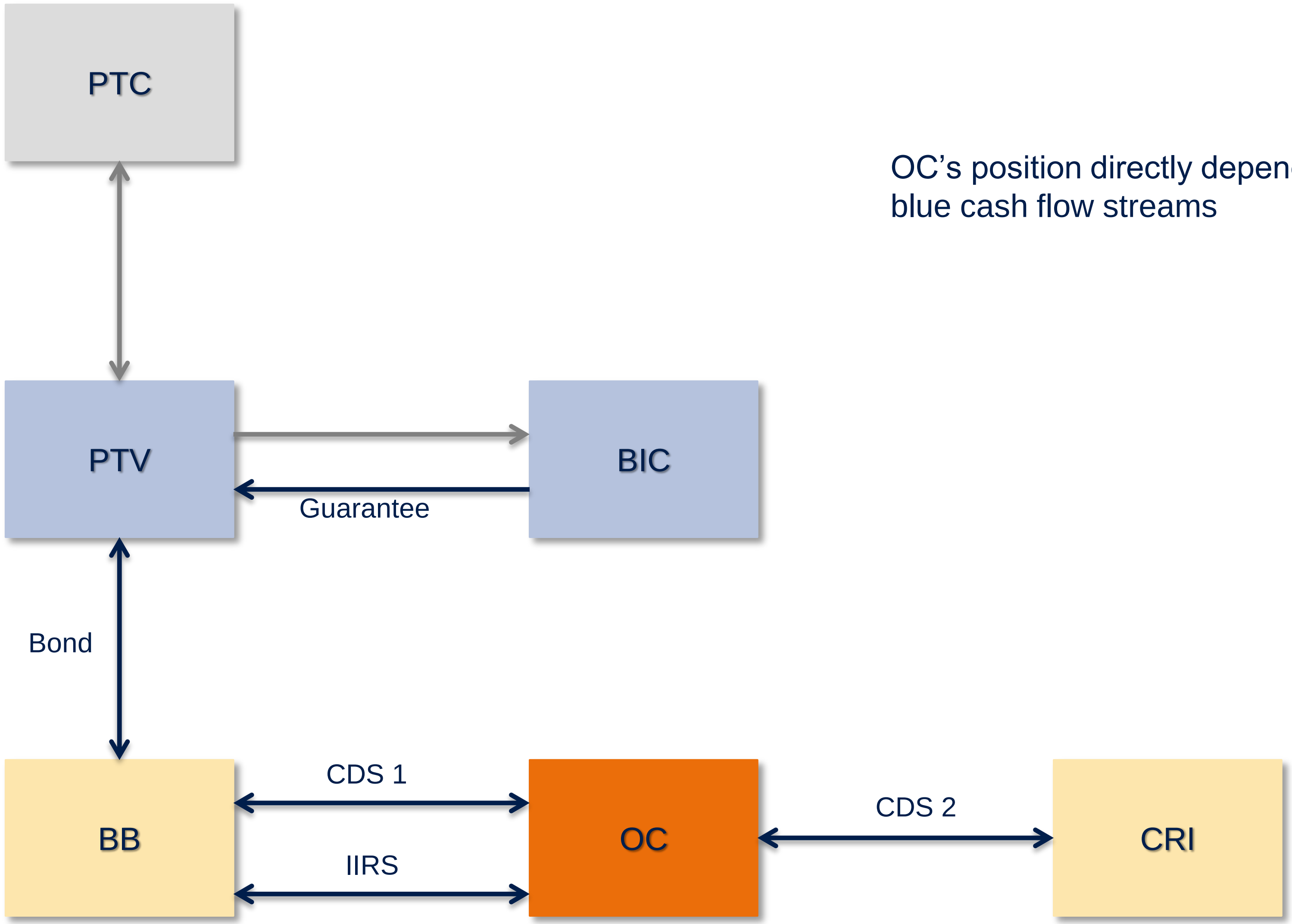
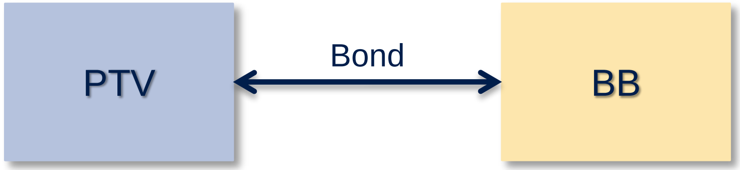
Overview over party‘s business relationships



OC’s position directly depends on all dark blue cash flow streams

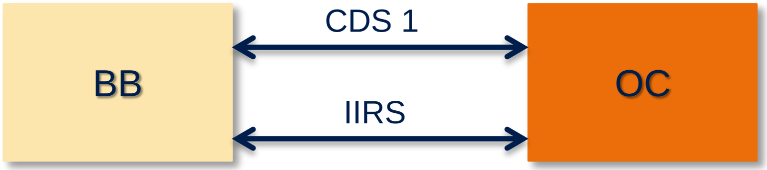
- BIC guarantees payments of bonds issued by PTV
- If PTV defaults before BIC…
  - After restructuring, PTV may pay a recovery amount or issue new bond with longer maturity and/or lower nominal (haircut)
  - BIC gets hold of the recovery payment (or new bond issues) and may choose to either pay the outstanding amount to the bond holder (suffering an immediate loss, the Loss Given Default (LGD)) or pay the scheduled regular coupon payments a if no default has occurred (pay-as-you-go, PAUG)
  - if BIC defaults later, the bond holder may receive another recovery payment from BIC
- If BIC defaults before PTV…
  - the bond holder will receive nothing from BIC
  - if PTV defaults later, the bond holder receives a recovery payment from PTV

- PTV pays some fees to BIC as compensation for giving the guarantee
  - This payment will not be considered since it has no effect on OC



- At start, BB pays PTV the initial bond notional and receives the bond
- The nominal of the bond increases with the inflation index
- The bond pays a regular coupon (i.e. quarterly) on the inflation indexed nominal based on a fixed rate
- Because of the variable bond notional, the coupons vary, too
- If PTV defaults, BB receives recovery or coupon payments from BIC
  - see previous slide

- BB faces inflation, interest and default risk
  - BB seeks to hedge this risk and end up with a default risk-free synthetic floater (i.e. variable interest payments plus spread, a riskless investment



- OC is the hedging partner of BB
- BB enters into two swap deals with OC
- First Deal: Credit Default Swap (CDS 1)
  - BB pays a premium to OC, until both, BIC and PTV default
  - In case of defaults of BIC and PTV, OC get holds of the remaining's of the bond issue (see above)
  - If both, BIC and PTV, are defaulted, OC has the option to choose PAUG or pay BB the LGD (physical settlement, PS)
- Second Deal: Inflation linked Interest Rate Swaps IIRS
  - BB passes over the full coupon from the bond to OC (notional is inflation indexed)
  - BB receives variable interest rate payments based on LIBOR from OC (notional is not inflation indexed)
  - If nominal of the IIRS decreases with any unscheduled early prepayments of the bond (e.g. recovery payments)
  - If BIC and PTV default and OC chooses PS, the IIRS terminates
- Both deals are collateralized
  - Credit default risk of BB and OC can be neglected in first order approximation

- OC buys protection from CRI against default of BIC and PTV to hedge default risk
- Credit Default Swap (CDS 2)
  - OC pays a premium to CRI, until both, BIC and PTV default
  - In case of defaults of BIC and PTV, CRI get holds of the remaining's of the bond issue (see above)
  - If both, BIC and PTV, are defaulted, CRI has the option to choose PAUG or pay OC the LGD (physical settlement, PS)
- If CRI defaults before BIC and PTV, CDS 2 terminates
  - If CDS 2 is in-the-money, OC may receive some recovery payments
- If CRI defaults after BIC and PTV, CDS 2 may continue with PAUG
  - OC receives recovery in terms of netted payments from some trusted account

Risk factors involved (from OC's perspective)

- Risk of default of three different counterparties
  - Risky parties are PTV, BIC, CRI
- Credit migration risk
  - Default of PTV may decrease credit worthiness of BIC
  - Default of PTC or BIC may decrease credit worthiness of CRI
- Interest rate risk
  - Due to fixed payments of bonds and future discounting
- Inflation risk
  - Bond nominal increases with inflation rate
- Recovery risk
  - The amount of recovery payments is uncertain

- Further risks (not modelled stochastically)
  - Funding cost (we assume deterministic funding costs)
  - Liquidation procedure (we do not know, how and when recovery payments will be paid)

Sources of information

» Official legal documentation of issued bonds
  - Often 200-500 pages of lawyers talk
  - Explicit, definite, clear mathematical formulas are usually avoided
  - Important details of the cash flow structure often have to be re-constructed by digging recursively into the glossary of the bond prospect
  - Sometimes, important information are far from being clear and unequivocal

» Term sheets of CDS contracts
  - Simplified contract for CDS deals, containing (most often) all details relevant for valuation of the contract
  - Refers to some standard ISDA framework agreement

» Market experience
  - Some important aspects can not been found in the contract
  - Example: After a default of the issuer the actual recovery payments depend on decisions of the liquidator, e.g. by newly issued bonds with longer maturities and reduced nominal (haircut).

Arbitrage free pricing

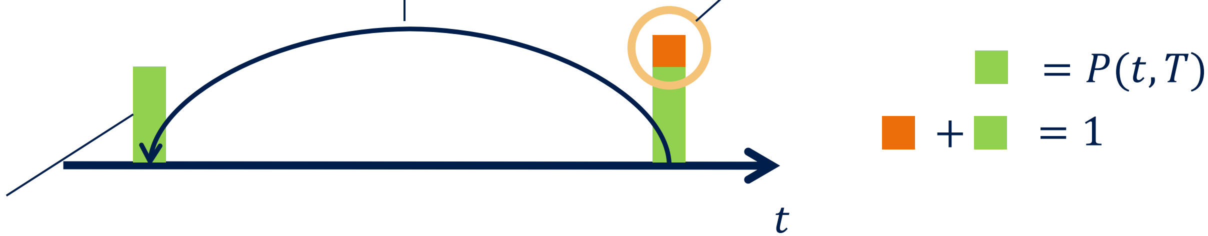
The basics

Time is money. But how much money is it?

- Money today is worth more than the same amount in some distant future
  - Risk of default
  - Missing earned (risk free) interest
- Zero discount bond $P(t,T)$: discounted value as of time t of payment of 1 unit at future time T
- Discount factor: Factor to be multiplied to a future cash flow to get its present value
  - Equal to price of zero bond

Discounting

Interest earned over
time period T:
$r\,(T-t)\,P(t,T)$



Cash flow 1 at future
time T is worth now
$P(t,T) = \frac{1}{1+r(T-t)}$

Compounding of interest rates

» Usually, interest is paid on a regular basis, e.g. monthly, quarterly or annually

» If re-invested, the compounding effect is significant

› Annual, semi-annual, quarterly, monthly or daily compounding is used

› Anyway, the rate r is quoted as annualised rate, i.e. interest per year

» Without re-investing, the rate is called “simple compounding”

$$\underbrace{\left(1+\frac{r}{m}\right)\left(1+\frac{r}{m}\right)\cdots\left(1+\frac{r}{m}\right)}_{nm\text{ times}} = \left(1+\frac{r}{m}\right)^{nm}$$

Compoundings per year

Number of years

» Continuous compounding is the limit of compounding in infinitesimal short time periods

$$\lim_{\substack{m\to\infty \\ n=\text{const.}}} \left(1+\frac{r}{m}\right)^{nm} = e^{rT}, \quad T=n, \qquad P(t,T)=e^{-rT}$$

If not stated otherwise, this convenient notation is used throughout the rest of the talk. It is also most often assumed in papers on finance.

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Arbitrage: Making money out of nothing

- Arbitrage is the art of earning money (immediately) without taking risk
- If the markets are inefficient, there may be opportunities for arbitrage
- Since money earned by arbitrage is easy money, market participants will take immediate advantage of arbitrage opportunities
- Fair values should be arbitrage free

- Example:
  - Party A offers to sell stock for 10 (ask price)
  - Party B is willing to buy stock for 15 (bid price)
  - Arbitrage! Buy from A and sell to B without risk, making riskless profit of 5
  - Because of this, A will have a lot of potential buyers (and will rise the price) while B has many offers and lowers the price, reducing the arbitrage opportunity until ask price > bid price

There is no free lunch!

Example: Valuation of forward contract

- Forward contract buy some asset (e.g. a stock) for a fixed price K at a later time T
  - Question: What is the fair strike price K?
- The bank replicates the Forward contract:

At time $t = 0$:

1. Step: Enter into short forward contract (0 cost)
2. Step: Borrow amount S_0 at risk free rate
3. Step: Buy stock at price S_0

At time $t = T$:

1. Step: Settle Forward contract and receive K in return for stock
2. Step: Pay back loan

Time	Forward contract on stock	Stock	Loan	Sum
$t = 0$	0	S_0	$-S_0$	0
$t = T$	$K - S_T$	S_T	$-e^{rT}S_0$	$K - e^{rT}S_0$

Assume $K > e^{rT}S_0$. In this case, our strategy has provided us with a riskless profit (> 0) at no cost, which contradicts the no-arbitrage assumption.

Assume $K < e^{rT}S_0$. In that case, use the opposite strategy: long forward contract, short sell stock and lend cash. You make $K - e^{rT}S_0$ out of zero investment, no risk.

To avoid arbitrage, the forward price (no dividends) must be $K = e^{rT} S_0$

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The fundamental theorem of asset pricing

Theorem: Suppose we have an arbitrage free market and a numeraire, i.a. an asset N with strictly positive price for all $t \in [0, T]$.

Then there exists a measure Q_N (the martingale measure) such that for any derivative V with payoff $V(T)$ the present value is given by

$$\frac{V(0)}{N(0)} = E_{Q_N}\left(\frac{V(T)}{N(T)}\right)$$

Example: With the zero bond $P(t, T)$ as numeraire, we get

$$V_{\text{Forward}}(0) = P(0, T)(E_T(S_T) - K)$$

and therefore

$$V_{\text{Forward}}(0) = S_0 - e^{-rT} K$$

since $S(t)$ follows $dS_t = rS_t dt + \sigma dW_t$ under the martingale measure associated to $P(t, T)$

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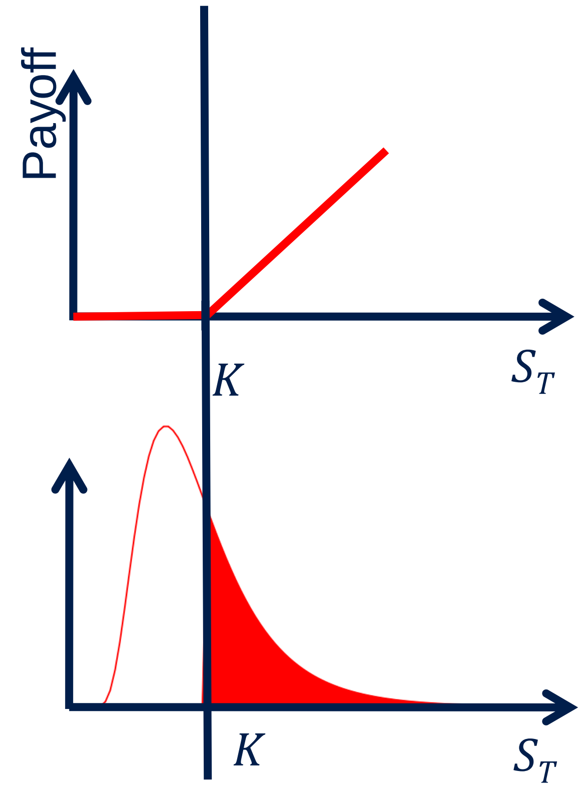
Adding optionality

- » For options, the distribution function matters
- » Plain Vanilla option: cut off distribution function at strike K
- » European Call option payoff: $\max(S_T - K, 0) \equiv (S_T - K)^+$

- » Question: Is there any arbitrage free replication strategy to finance these payoffs?



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Stock process

The Geometric Brownian motion of some stock price $S(t)$

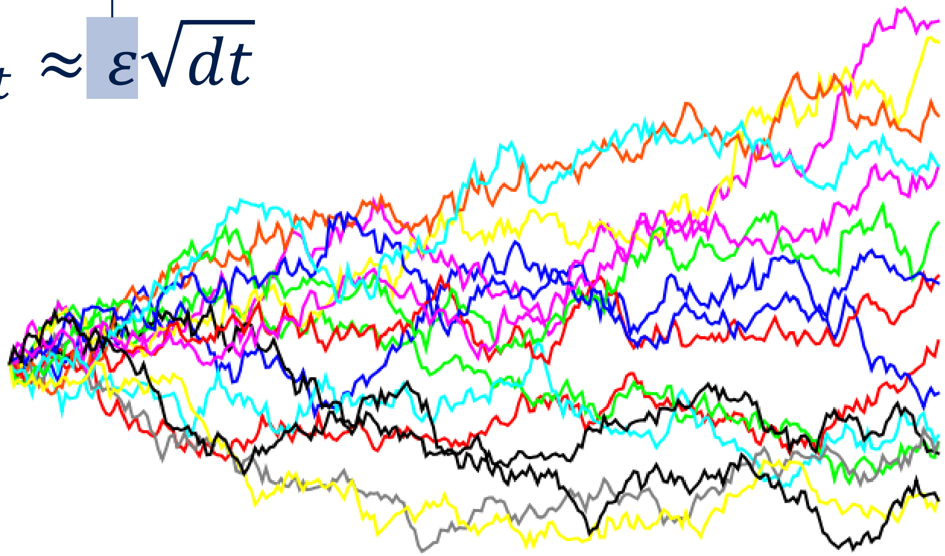
Drift

Volatility

Standard normal distributed random number

$$dS_t = \mu S_t dt + \sigma S_t dW_t \qquad dW_t \approx \varepsilon\sqrt{dt}$$

$$d \ln S_t = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW_t$$



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Ito‘s lemma

» What’s the stochastic process of a function of a stochastic process?
  › Apply Ito’s lemma

» Process of underlying: $dS = \mu S dt + \sigma S dW$

» Fair value V of option is function of S: $V = V(S)$

» Ito‘s lemma:

$$dV = \left(\frac{\partial V}{\partial S}\mu S + \frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial V}{\partial S}\sigma S dW$$

Caused by stochastic term $\sim\sqrt{dt}$.

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Replication Portfolio for general claims

- » Replicate option payoff by holding portfolio of cash account and stock
- » Ansatz: $V = B + xS$ with $dB = rBdt$.

- » Changes in option fair value V

$$dV = \left(\frac{\partial V}{\partial S}\mu S + \frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial V}{\partial S}\sigma S dW$$

$$dV = rBdt + x\mu Sdt + x\sigma SdW$$

- » Choose $x = \frac{\partial V}{\partial S}$ and insert for $B = V - xS$:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 = rB = rV - rS\frac{\partial V}{\partial S}$$

$$\Leftrightarrow \quad \frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 = rV$$

Black-Scholes PDE

With this choice of x, the stochastic term vanishes

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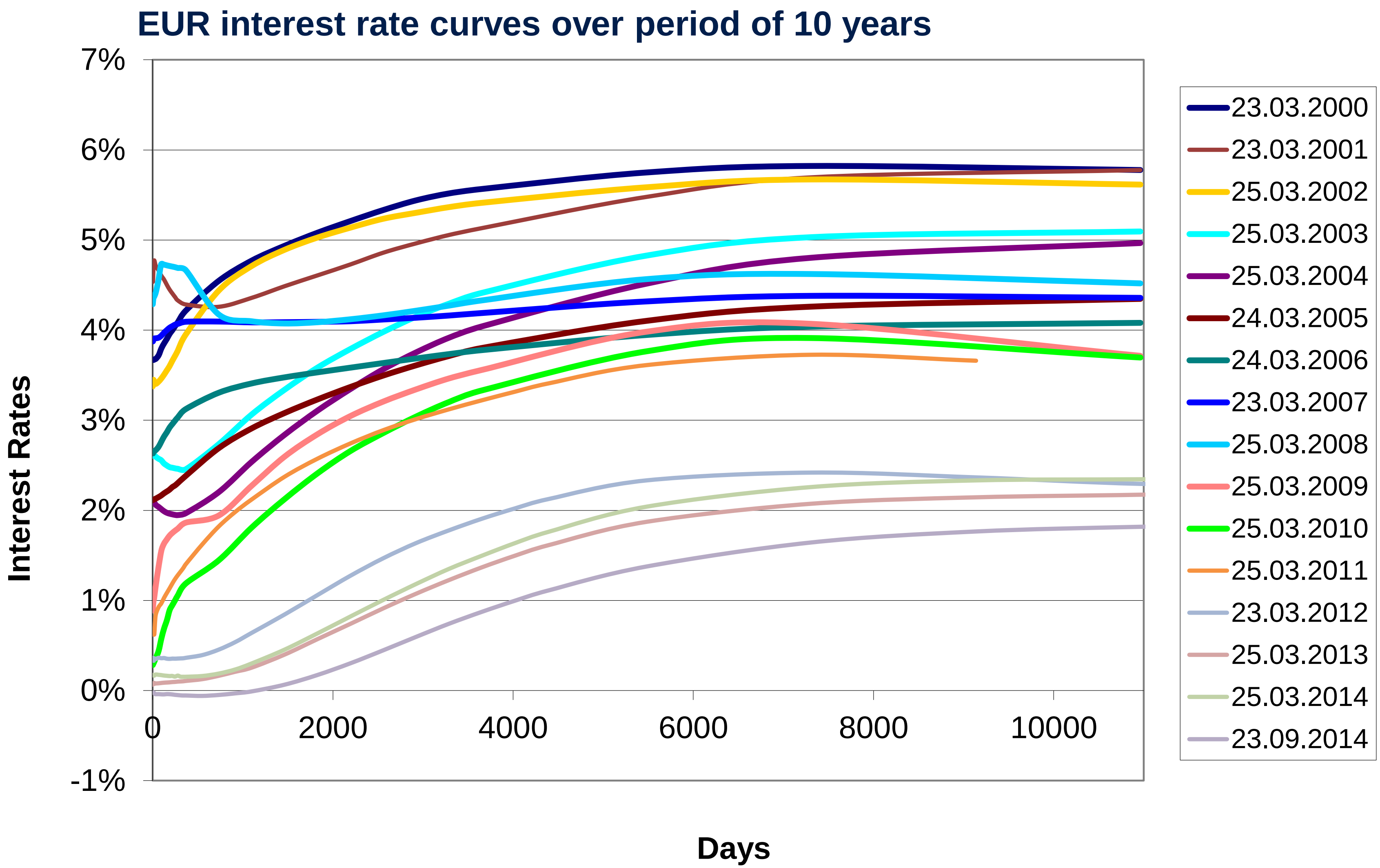
The model

Get the price right

Historical interest rate curves



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Modelling interest rates

» Choice of model is Heath-Jarrow-Morton-like model with Heston-like stochastic volatility[1]

› Total number of stochastic factors is $2N$

› Interest rate curve is modelled in terms of instantaneous forward rate $f(t,T)$, i.e. the interest rate for the infinitesimal short future time period from T to $T+dt$ as seen from time t.

$$df(t,T) = \mu_f(t,T)dt + \sum_{i=1}^{N} \sigma_{f,i}(t,T)\sqrt{v_i(t)}dW_i^Q(t) \qquad i = 1,2,\dots,N$$

$$dv_i(t) = \kappa_i(\theta_i - v_i(t))dt + \sigma_i\sqrt{v_i(t)}\left(\rho_i dW_i^Q(t) + \sqrt{1-\rho_i^2}dZ_i^Q(t)\right)$$

› Absence of arbitrage implies the following drift term:

$$\mu_f(t,T) = \sum_{i=1}^{N} v_i(t)\sigma_{f,i}(t,T)\int_t^T \sigma_{f,i}(t,u)du$$

› $\sigma_{f,i}(t,T)$ is an arbitrary function, but we will choose the convenient, but still flexible form

$$\sigma_{f,i}(t,T) = \left(\alpha_{0,i} + \alpha_{1,i}(T-t)\right)e^{-\gamma_i(T-t)}$$

› In the following, we set $N = 1$

1 A. B. Trolle and E. S. Schwartz, *A General Stochastic Volatility Model for the Pricing of Interest Rate Derivatives*, The Review of Financial Studies / v 22 n 5 2009

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Semi-Analytic discount factors

- Discount factors (or zero bonds) are given by $P(t,T) = e^{-\int_t^T f(t,u)du}$, which can be solved analytically

$$P(t,T) = \frac{P(0,T)}{P(0,t)} e^{B(T-t)x(t) + \sum_{j=1}^{6} B_j(T-t)\phi_j(t)}$$

- State variables given by set of ODEs

$$dx(t) = -\gamma x(t)dt + \sqrt{v(t)}dW^Q(t)$$
$$d\phi_1(t) = (x(t) - \gamma\phi_1(t))dt$$
$$d\phi_2(t) = (v(t) - \gamma\phi_2(t))dt$$
$$d\phi_3(t) = (v(t) - 2\gamma\phi_3(t))dt$$
$$d\phi_4(t) = (\phi_2(t) - \gamma\phi_4(t))dt$$
$$d\phi_5(t) = (\phi_3(t) - 2\gamma\phi_5(t))dt$$
$$d\phi_6(t) = (2\phi_5(t) - 2\gamma\phi_6(t))dt$$

$$B(\tau) = \frac{\alpha_1}{\gamma}\left(\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1}\right)(e^{-\gamma\tau} - 1) + \tau e^{-\gamma\tau}\right)$$
$$B_1(\tau) = \frac{\alpha_1}{\gamma}(e^{-\gamma\tau} - 1)$$
$$B_2(\tau) = \left(\frac{\alpha_1}{\gamma}\right)^2\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1}\right)\left(\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1}\right)(e^{-\gamma\tau} - 1) + \tau e^{-\gamma\tau}\right)$$
$$B_3(\tau) = -\frac{\alpha_1}{\gamma^2}\left(\left(\frac{\alpha_1}{2\gamma^2} + \frac{\alpha_0}{\gamma} + \frac{\alpha_0^2}{2\alpha_1}\right)(e^{-\gamma\tau} - 1) + \left(\frac{\alpha_1}{\gamma} + \alpha_0\right)\tau e^{-2\gamma\tau} + \frac{\alpha_1}{2}\tau^2 e^{-2\gamma\tau}\right)$$
$$B_4(\tau) = \left(\frac{\alpha_1}{\gamma}\right)^2\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1}\right)(e^{-\gamma\tau} - 1)$$
$$B_5(\tau) = -\frac{\alpha_1}{\gamma^2}\left(\left(\frac{\alpha_1}{\gamma} + \alpha_0\right)(e^{-\gamma\tau} - 1) + \alpha_1\tau e^{-2\gamma\tau}\right)$$
$$B_6(\tau) = -\frac{1}{2}\left(\frac{\alpha_1}{\gamma}\right)^2(e^{-\gamma\tau} - 1)$$

$$x(0) = \phi_1(0) = \phi_2(0) = \phi_3(0) = \phi_4(0) = \phi_5(0) = \phi_6(0) = 0$$

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Free model parameter

Parameter	Description
α_0	Short-term volatility limit
α_1	Volatility time scaling factor
γ	Volatility damping factor
$v_0 = v(0)$	Initial variance
κ	Variance drift
θ	Variance mean reversion
σ	Volatility of variance
ρ	Correlation between $dW^Q(t)$ and $dZ^Q(t)$

One redundant parameter out of α_0, α_1, κ, and σ, leaving 7 independent parameters

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Zero bond option prices

» Pay off of European put option with strike K with expiry at T_0 on zero bond with maturity T_1

$$V_{\mathrm{ZBPUT}}(T_0, T_0, T_1, K) = \left(K - P(T_0, T_1)\right)^+$$

» Price as of time $t < \mathrm{T}_0$ is given by

$$V_{\mathrm{ZBPUT}}(t, T_0, T_1, K) = K G_{0,1}(\ln(K)) - G_{1,1}(\ln(K))$$

with

$$G_{a,b}(y) = \frac{\psi(a, t, T_0, T_1)}{2} - \frac{1}{\pi} \int_0^\infty \frac{\mathrm{Im}\left[\psi(a + iub, t, T_0, T_1) e^{-iuy}\right]}{u} du$$

and

$$\psi(u, t, T_0, T_1) = E_t^Q\left[e^{-\int_t^{T_0} r_s ds} e^{u \ln(P(T_0, T_1))}\right]$$

is the Fourier transform of the expected zero bond price.

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Solution of Fourier transform

» The Fourier transform can be expressed as a system of ODE's

$$\psi(u,t,T_0,T_1) = \exp\Big(M(T_0 - t) + N(T_0 - t)v(t) + u\ln\big(P(t,T_1)\big) + (1-u)\ln(P(t,T_0))\Big)$$

with

$$\frac{dM(\tau)}{d\tau} = N(\tau)\kappa\theta$$

$$\frac{dN(\tau)}{d\tau} = N(\tau)\Big(-\kappa + \sigma\rho\big(uB(T_1 - T_0 + \tau) + (1-u)B(\tau)\big)\Big)$$

$$+\frac{1}{2}N^2(\tau)\sigma^2 + \frac{1}{2}(u^2 - u)B^2(T_1 - T_0 + \tau)$$

$$+\frac{1}{2}\big((1-u)^2 - (1-u)\big)B^2(\tau)$$

$$+u(1-u)B(T_1 - T_0 + \tau)B(\tau)$$

and $M(0) = N(0) = 0$.

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Approximating swaption prices

» Swaption pay off is given by

$$V_{\text{Swaption}}(T_m, T_m, T_n, K) = \left(1 - P(T_m, T_n) - K \sum_{j=m+1}^{n} \left(T_j - T_{j-1}\right) P(T_m, T_j)\right)^+$$
$$= \left(1 - P(T_m, T_n) - KA(T_m, T_n, t)\right)^+$$

» Approximating swaption prices by zero bond prices

$$V_{\text{Swaption}}(t, T_m, T_n, K) = \zeta V_{\text{ZBPUT}}(t, T_m, t + D(t), \zeta^{-1}) \qquad \zeta = \frac{KA(T_m, T_n)}{P(t, t + D(t))}$$

» Find $D(t)$, such that swap and zero bond have same relative volatility:

$$B^2(D(t)) = \left(\sum_{j=m+1}^{n} \frac{\left(T_j - T_{j-1}\right) P(t, T_j)}{A(T_m, T_n)} B(T_j - t)\right)^2$$

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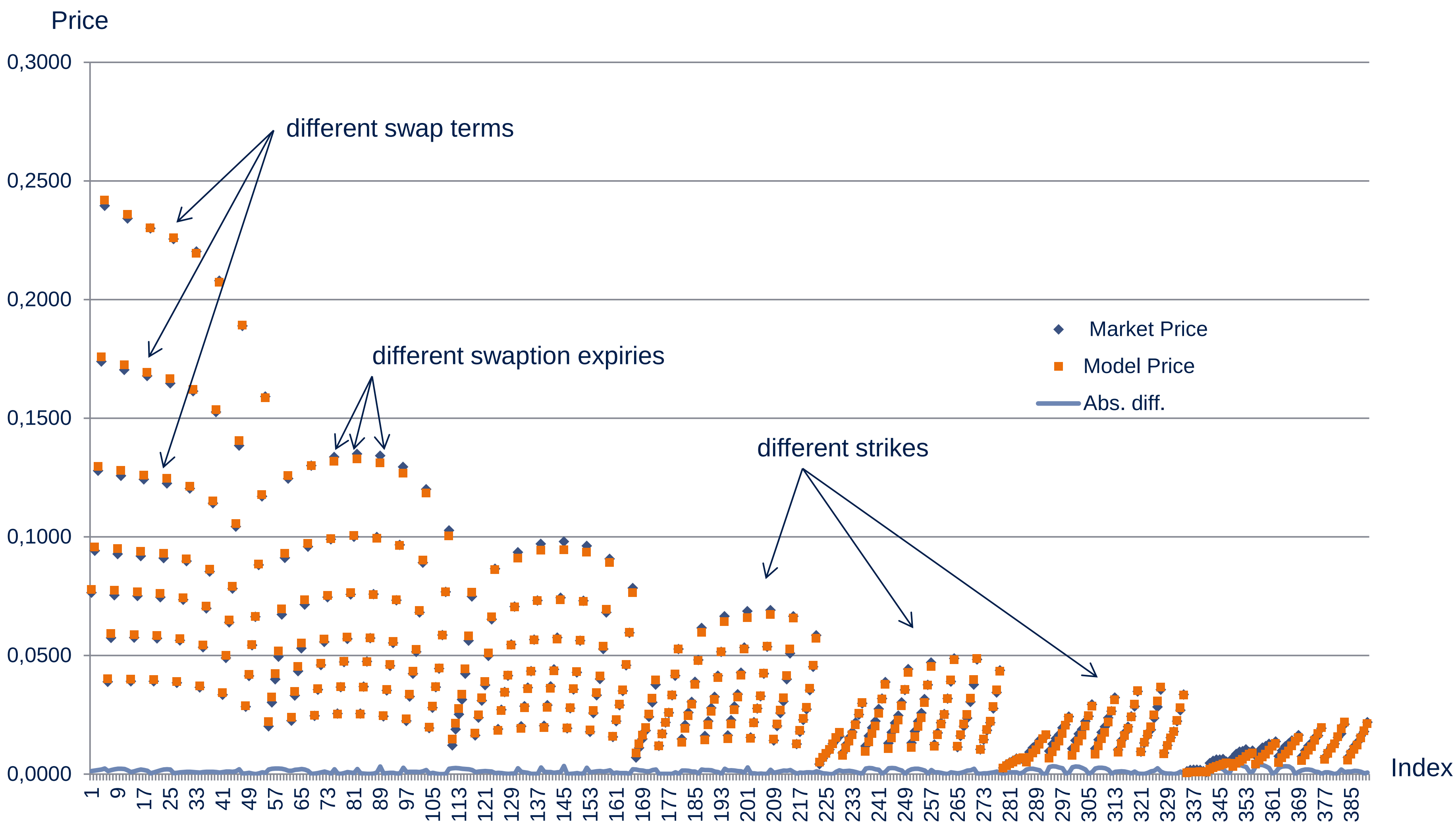
Calibration check against swaptions



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Modelling inflation

» Extension of Jarrow-Yildirim model

- Inflation rate at any time point is the difference between nominal rates (the observed interest rate) and real (but hidden) rates, but with stochastic volatility

$$d\ln\big(I(t,T)\big) = \left(n(t) - r(t) - \frac{1}{2}v_I(t)\right)dt + \sqrt{v_I(t)}\,dW_I^{Q_n}(t)$$

$$dv_I(t) = \kappa_I(\theta_I - v_I(t))dt + \sigma_I\sqrt{v_I(t)}\left(\rho_I dW_I^{Q_n}(t) + \sqrt{1-\rho_I^2}\,dZ_I^{Q_n}(t)\right)$$

- Nominal and real rate are given in terms of zero bonds

$$n/r\,(T) = -\lim_{T\to\infty}\frac{\partial \ln P_{n/r}(t,T)}{\partial T}$$

- Thus, we need to model two interest rate models!

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Change of numeraire

» The processes for state variable $x_n(t)$ and variance $v_n(t)$ need to be adjusted for change of numeraire to nominal risk neutral measure

$$dx_r(t) = -\gamma_r\left(x_r - \rho_{I,x_r}\sqrt{v_r v_I}\right)dt + \sqrt{v_r(t)}dW_r^{Q_n}(t)$$

$$dv_r(t) = \left(\kappa_r(\theta_r - v_r(t)) - \sigma_r\rho_{I,x_r}\sqrt{v_r v_I}\right)dt + \sigma_r\sqrt{v_r(t)}\left(\rho_r dW_r^{Q_n}(t) + \sqrt{1-\rho_r^2}dZ_r^{Q_n}(t)\right)$$

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Full hybrid inflation/interest-rate model

$$dx_n(t) = -\gamma_n x_n(t)dt + \sqrt{v_n(t)}dW_n^{Q_n}(t)$$
$$dv_n(t) = \kappa_n(\theta_n - v_n(t))dt + \sigma_n\sqrt{v_n(t)}\left(\rho_n dW_n^{Q_n}(t) + \sqrt{1-\rho_n^2}dZ_n^{Q_n}(t)\right)$$

$$dx_r(t) = -\gamma_r\left(x_r - \rho_{I,x_r}\sqrt{v_r v_I}\right)dt + \sqrt{v_r(t)}dW_r^{Q_n}(t)$$
$$dv_r(t) = \left(\kappa_r(\theta_r - v_r(t)) - \sigma_r\rho_{I,x_r}\sqrt{v_r v_I}\right)dt + \sigma_r\sqrt{v_r(t)}\left(\rho_r dW_r^{Q_n}(t) + \sqrt{1-\rho_r^2}dZ_r^{Q_n}(t)\right)$$

$$d\ln\big(I(t,T)\big) = \left(n(t) - r(t) - \frac{1}{2}v_I(t)\right)dt + \sqrt{v_I(t)}dW_I^{Q_n}(t)$$
$$dv_I(t) = \kappa_I(\theta_I - v_I(t))dt + \sigma_I\sqrt{v_I(t)}\left(\rho_I dW_I^{Q_n}(t) + \sqrt{1-\rho_I^2}dZ_I^{Q_n}(t)\right)$$

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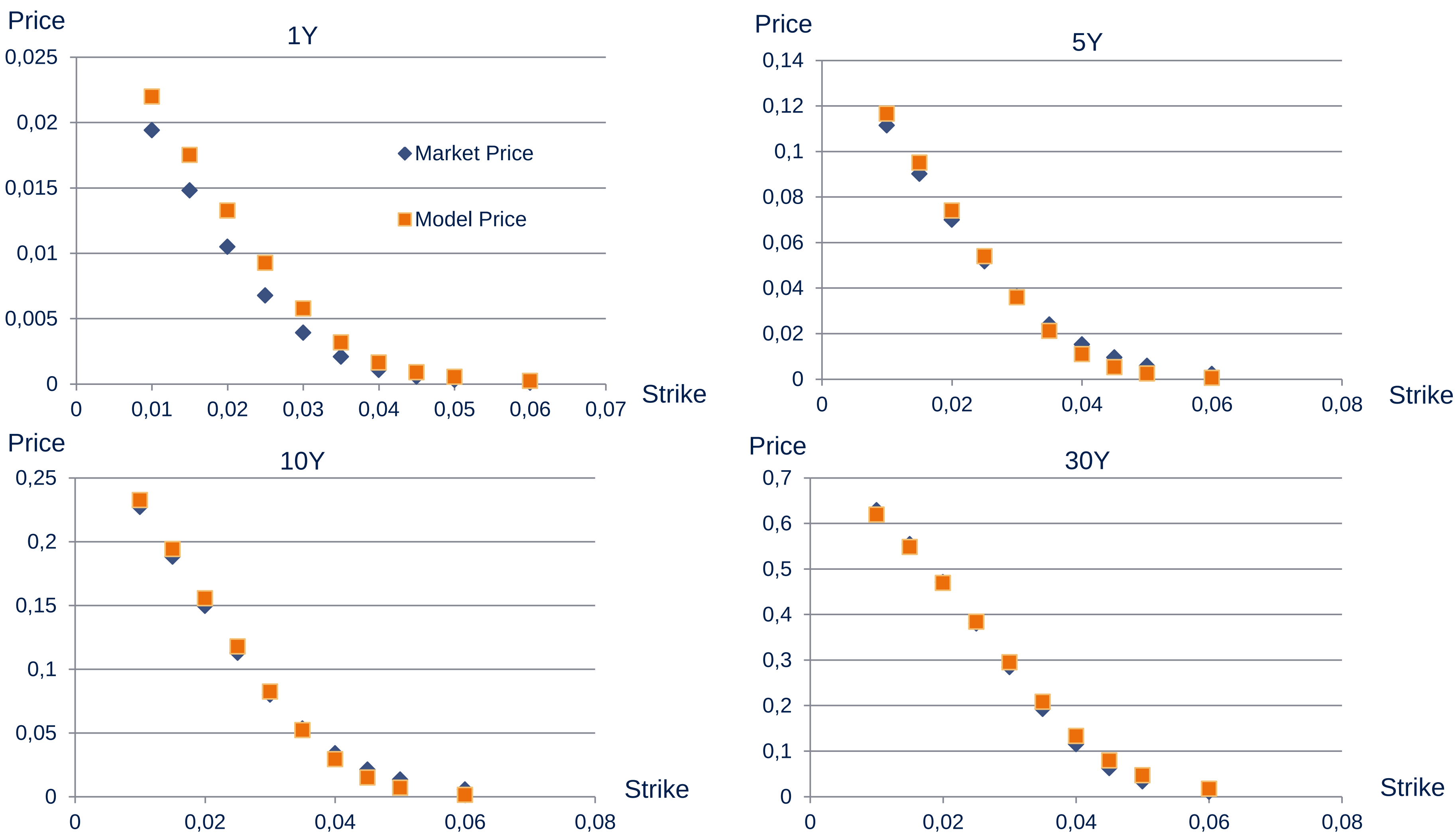
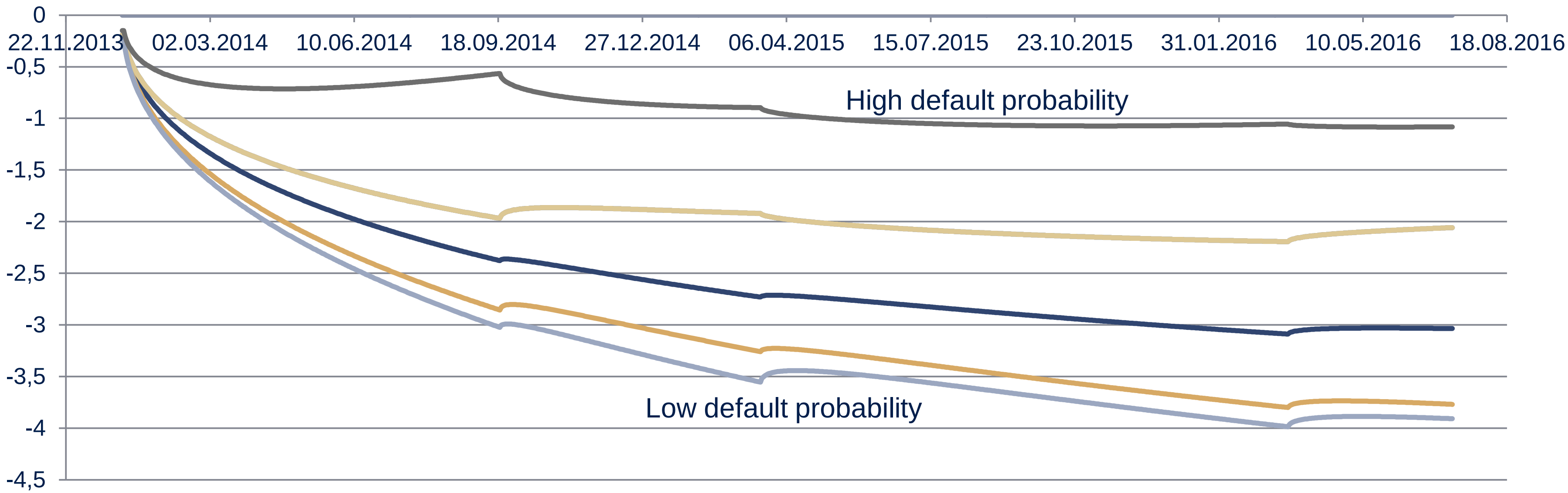
Calibration check against inflation caps



Firm‘s value model for modelling default risk

- Define a normally distributed “firm’s value process” $X(t)$ with $dX(t) = dW^Q(t)$.
- $X(t)$ is normally distributed with mean 0 and standard deviation $\sqrt{t}$.
- Define a lower boundary $K(t)$ (threshold curve), such that if $X(t)$ falls below $K(t)$, $X(t) \leq K(t)$, than the entity defaults
- The boundary function $K(t)$ is calibrated to some exogenously given default probability curve
- Example of calibrated threshold curves:



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Which survival probability to choose?

» Two (main) sources of default probability curves
  › Implied from liquid credit default swap (CDS) quotes
  › Estimated from historical time series with respect to rating, industry sector, currency, and/or country

Implied from CDS quotes	Historical probabilities
+ Reflect price for hedging against default risk + Quotes frequently updated in immediate response to new information + Consistent with standard CDS pricing + More specific to certain entities – Include components unrelated to default probability, i.e. liquidity spreads – As a consequence, highly overestimated probability of default events – Implied forward default probabilities may become negative	+ More realistic probability of default events + Directly available (no bootstrapping required) + Pure default probability free from any other effects – Based on infrequently updated ratings, does reflect credit quality changes with large time delay – Inconsistent default probabilities from different rating agencies – Application to arbitrage free pricing requires calibration of unobservable parameters (i.e. liquidity spread)

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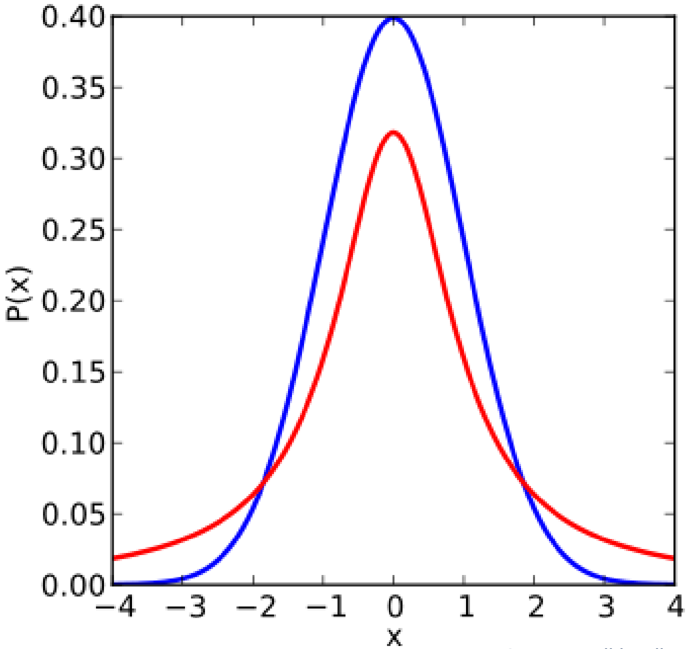
Modelling of recovery rates

» Recovery rate: percentage of outstanding (risk-free) PV that can be recovered after a default
  › Often assumed to be a fixed amount
  › Alternative definition: percentage of outstanding nominal

Blue: standard normal distribution
Read: student‘s-t distribution



Source: Wikipedia

» Assumption: recovery rate has student’s-t distribution:

$$R \sim \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\sigma\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}}$$

with one degree of freedom ($\nu = 1$) and free parameters mean μ and scaling factor σ.

» Recovery is cut off at 0 and 1

Correlation matrix

	x_n	v_n	x_r	v_r	I	v_I	X_{PTV}	X_{BIC}	X_{CRI}
x_n	1	0.124	0.918	0.137	-0.2353	-0.0279	0	0	0
v_n	0.124	1	0.137	0.99	0	0	0	0	0
x_r	0.918	0.137	1	0.124	-0.4495	0.15972	0	0	0
v_r	0.137	0.99	0.124	1	0	0	0	0	0
I	-0.235	0	-0.450	0	1	-0.652	0	0	0
v_I	-0.028	0	0.160	0	-0.652	1	0	0	0
X_{PTV}	0	0	0	0	0	0	1	0.1	0.3
X_{BIC}	0	0	0	0	0	0	0.1	1	0.3
X_{CRI}	0	0	0	0	0	0	0.1	0.3	1

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Regime switching

» Default of PTV has worsening impact on BIC and CRI
  › Additional obligations put on already stressed entities
  › Credit quality worsening
  › Credit rating downgrade possible
» Similar effect of BIC default on CRI

» This Effect is models by means of a regime switching approach
  › In case of a default event of PTV and/or BIC, the threshold curve of all firm’s value models of entities, which depend on PTV and/or BIC, will be replaced by some threshold curve with higher default probability
  › This will increase implicitly the correlation between default events

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Modelling of PAUG option

- PAUG option: Credit guarantor or CDS protection seller may choose to either pay
  - Receive the bond from and pay the outstanding principal to the protection buyer (physical settlement)
  - Pay any difference between actual and contractual bond coupon and nominal payments when the actual payment is due (pay-as-you-go or PAUG)
- The valuation of the PAUG option requires to price the bond and all payments of the CDS at any time the PAUG option may be chosen.
- Because of its impact on the cash flow stream, funding cost of CDS protection seller needs to be considered
  - Payment of principal amount may require funding, if the bond could not be immediately sold in the market
- After default and restructuring, the remaining bond still as some default risk

What else?

- Many discussions with various people on client's side about the correct interpretation of the contract and how, if at all, they should be modelled
- Getting high-quality market data
  - Consistency and consistent Timing (especially when correlations should be calculated from historical data)
  - Availability (sometimes mapping to related data)
- Calibration
  - Very time consuming (e.g., if no analytical approximations are available)
  - Often unstable
- Additional features and complications not mentioned here (e.g. inflation seasonality)
- Implementing the MC code and testing, testing, testing
  - The 90/90-rule applies

- Documentation of results
  - Explicit reference to all assumptions
  - General rule: if it's not documented, it's not done

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